① Find any local and absolute extrema of
$f(x) = x^3 - 12x - 5$ algebraically.

② Discuss the slope of $y = x^3$ on the interval $(-\infty, \infty)$.
Be as detailed as you can.

rate of change at $x = 0$ is 0 $\qquad f'(x) = 3x^2$

~~$(-\infty, 0)$ rate of change neg~~

$(0, \infty)$ rate of change pos.

rate of change never negative

rate of change decreasing $(-\infty, 0)$
increasing $(0, \infty)$

$f(x) = x^3 - 12x - 5$

$f'(x) = 3x^2 - 12$

$0 = 3x^2 - 12$

$x = \frac{-0 \pm \sqrt{0^2 - 4(3(-12))}}{2(3)} = \frac{\pm\sqrt{144}}{6} = \boxed{\pm 2}$

$\lim_{x \to -2} -2^3 - (12(-2)) - 5 = 11$

$\lim_{x \to 2} 2^3 - 12(2) - 5 = -21$

-2 is local max

2 is local min

$f'(x) = 3x^2 - 12$

critical pts ± 2

$+ \quad\quad - \quad\quad +$

$-2 \quad\quad 2$

max $\quad\quad$ min

$f'(x) = 3x^2$

critical at $x = 0$

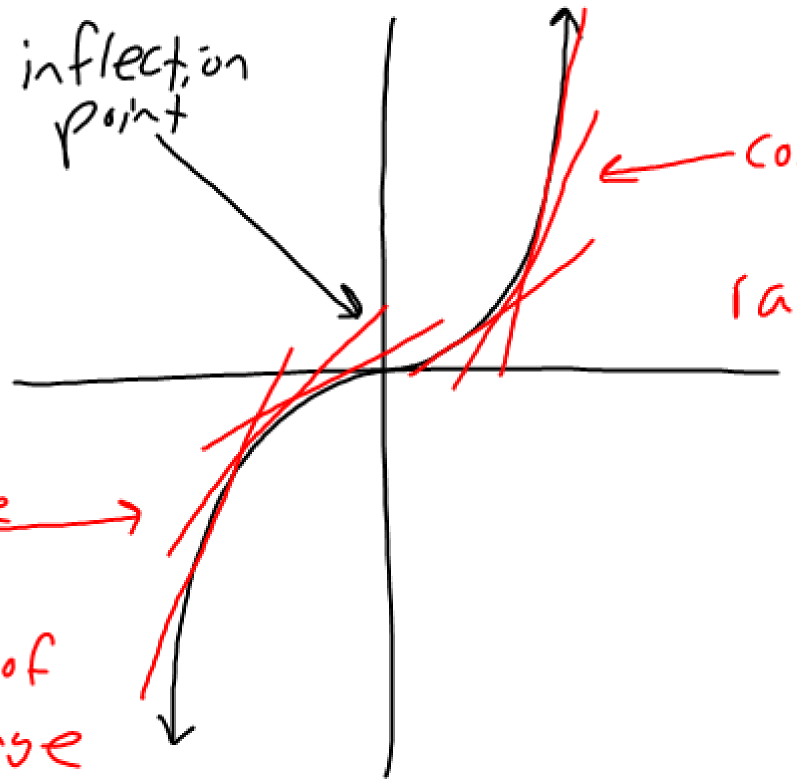
$+ \quad\quad +$

0

No extrema

Concavity

inflection point



concave up

rate of change increasing

$f''(x) > 0$

concave down

rate of change decreasing

$f''(x) < 0$

$f(x) = x^3$

$f'(x) = 3x^2$

$f''(x) = 6x$

$(-\infty, 0)$

$f''(x) < 0$

concave down

$(0, \infty)$

$f''(x) > 0$

concave up

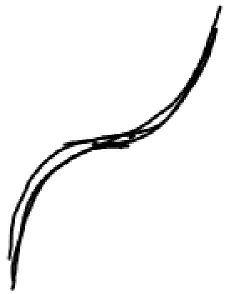
$x = 0$

inflection pt.

Warning

2^{nd} derivative can be zero at non-inflection points

check the pt. for a sign change



$f(x) = x^4$

$f'(x) = 4x^3$

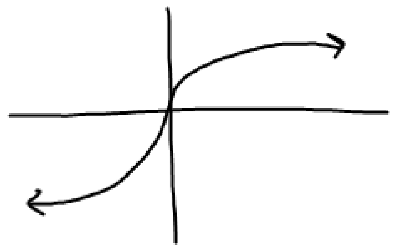
$f''(x) = 12x^2$ $\quad f''(x) = 0$ at $x = 0$

$+ \quad | \quad +$ No inflection

0

2^{nd} derivative could undefined at an inflection point

$\sqrt[3]{x}$



2nd derivative Test

if $f'(c) = 0$ and $f''(c) < 0$ have a maximum

if $f'(c) = 0$ and $f''(c) > 0$ have a minimum

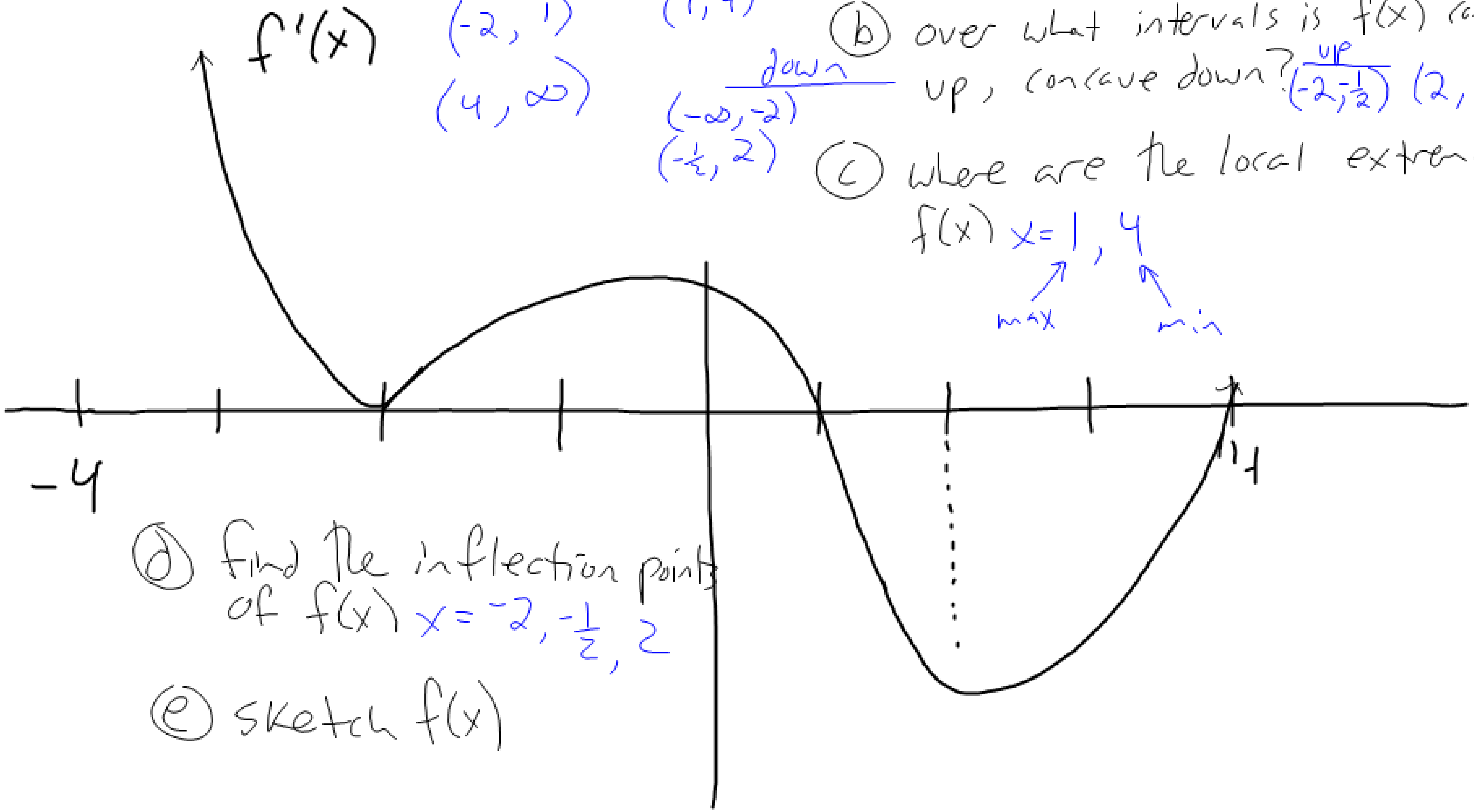
$f(x) = x^3 - 12x - 5$

$f'(x) = 3x^2 - 12$ critical pts at -2 & 2

$f''(x) = 6x$ $\quad f''(-2) = \text{neg}$ max $\quad f''(2) = \text{pos}$ min

$\boxed{\text{Warning}}$

The test fails if $f''(x) = 0$ or is undefined then you have to use first derivative test



increasing
$(-\infty, -2)$
$(-2, 1)$
$(4, \infty)$

decreasing
$(1, 4)$

a) Over what intervals is $f(x)$ increasing, decreasing?

b) over what intervals is $f(x)$ concave up, concave down?

up
$(-2, -\frac{1}{2})$ $(2, \infty)$

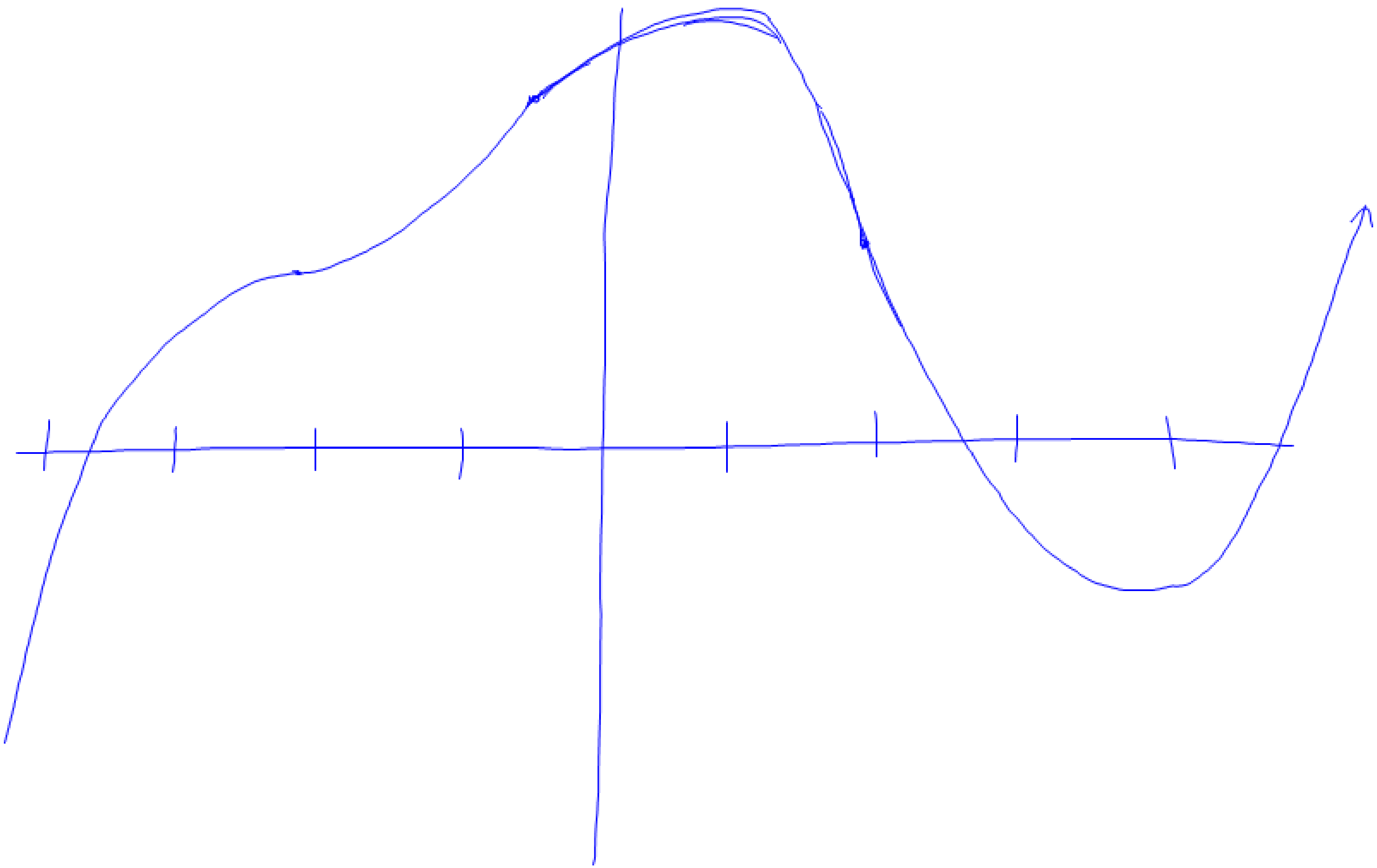
down
$(-\infty, -2)$
$(-\frac{1}{2}, 2)$

c) where are the local extrema of $f(x)$ $x = 1, 4$

max min

d) find the inflection points of $f(x)$ $x = -2, -\frac{1}{2}, 2$

e) sketch $f(x)$



$\boxed{\text{Summary}}$

$f(x)$ is increasing when $f'(x) > 0$

$f(x)$ is decreasing when $f'(x) < 0$

$f(x)$ has a critical point when
$f'(x) = 0$ or when $f'(x)$ is undefined

$f(x)$ is concave up when the rate
of change is increasing or $f''(x) > 0$

$f(x)$ is concave down when the rate
of change is decreasing or $f''(x) < 0$

$f(x)$ has critical points when $f'(x) = 0$
or is undefined

$f(x)$ has a possible inflection point
when $f''(x) = 0$ or is undefined

First Derivative test, test the sign
on either side of the critical points
a change in sign indicates an extrema

Second Derivative test, evaluate the second
derivative of $f(x)$ at the critical points,
if $f''(c) > 0$ then there is a minimum
if $f''(c) < 0$ then there is a maximum

$\boxed{4.3}$

#2, 6, 7, 10, 11, 14, 16, 19, 21, 24

No Calc	Calc	Take home
100 pts	50 pts	50 pts.